

* Defⁿ(3): A mapping $f: S \rightarrow T$ is called onto, if for each $t \in T$ $\exists$ some $s \in S$ such that $f(s) = t$.

* Defⁿ(4): Two mappings $f: S \rightarrow T$ and $g: S \rightarrow T$ are said to be equal

if,

$$f(x) = g(x) \quad \forall x \in S.$$

* Defⁿ(5) The product or Composite of two mappings $f: S \rightarrow T$ and $g: T \rightarrow U$ is defined as the mapping $g \circ f: S \rightarrow U$ such that $(g \circ f)(x) = g(f(x)) \quad \forall x \in S$.

Illustrations:

1. $f: \mathbb{N} \rightarrow \mathbb{N}$ defined as $f(x) = x^2 \quad \forall x \in \mathbb{N}$ is one-to-one, since for any $x, y \in \mathbb{N}$.

$$x \neq y \Rightarrow x^2 \neq y^2 \Rightarrow f(x) \neq f(y).$$

However, f is not onto,

since, for $2 \in \mathbb{N}$ $\nexists$ any $x \in \mathbb{N}$ such that $f(x) = 2$.

$$[\because f(x) = x^2 = 2 \Rightarrow x = \pm\sqrt{2} \notin \mathbb{N}]$$

(2.) $f: \mathbb{I} \rightarrow \mathbb{I}$ defined as $f(x) = x+3 \quad \forall x \in \mathbb{I}$ is 1-1 and onto;

Let $x, y \in \mathbb{I}$, then $f(x) = f(y)$

$$\Rightarrow x+3 = y+3$$

$$\Rightarrow x = y$$

i.e., $f(x) = f(y) \Rightarrow x = y$, $\therefore$ so f is 1-1

Now, let $x \in \mathbb{I}$ be arb. then

~~then~~ $(\exists y) \exists y \in \mathbb{I}$ s.t. $f(y) = x$.